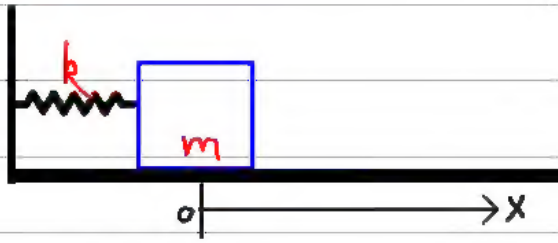
Meriam 8/2: Vibrations (contd.)

Basic Linear Vibrations



- Place origin at neutral position
- Displace mass a small +ve distance x
- Release the mass

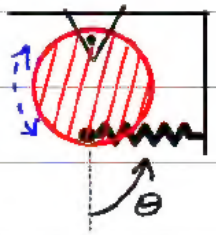
$$\Sigma F = -kx = m\ddot{x}$$

- Solution is of the form:

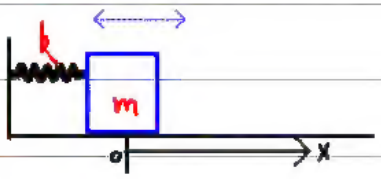
$$x(t) = \boxed{C_1 \sin(\omega_n t + \phi)}, \quad \omega_n = \sqrt{k/m}$$

These represent <u>one-dimensional</u>, <u>translational</u> vibrations.

How about rotational vibrations?



Translational



$$\Sigma F_x = -kx = ma_x = m\ddot{x}$$

$$\ddot{x} + \frac{k}{m}x = 0$$

$$\ddot{x} + \omega_n^2 x = 0, \quad \omega_n^2 = k/m$$

$$\boxed{x(t) = \text{Amplitude} \cdot \sin(\omega_n t + \phi)}$$

Rotational

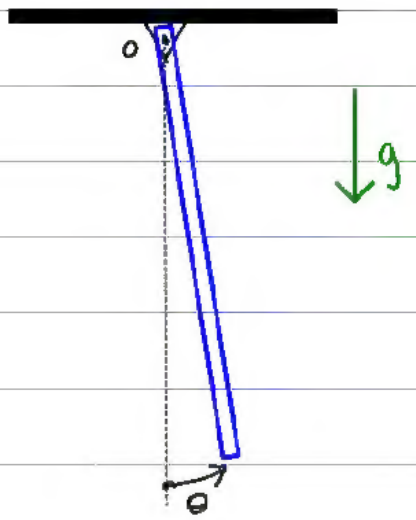
$$\Sigma M_o = -K\theta = I_o\alpha = I_o\ddot{\theta}$$

$$\ddot{\theta} + \frac{K}{I_o}\theta = 0$$

$$\ddot{\theta} + \omega_n^2\theta = 0, \quad \omega_n^2 = K/I_o$$

$$\boxed{\theta(t) = \text{Amplitude} \cdot \sin(\omega_n t + \phi)}$$

E.g. Bar pendulum.



Given: length L
mass m
small oscillation amplitude.

Find ω_n

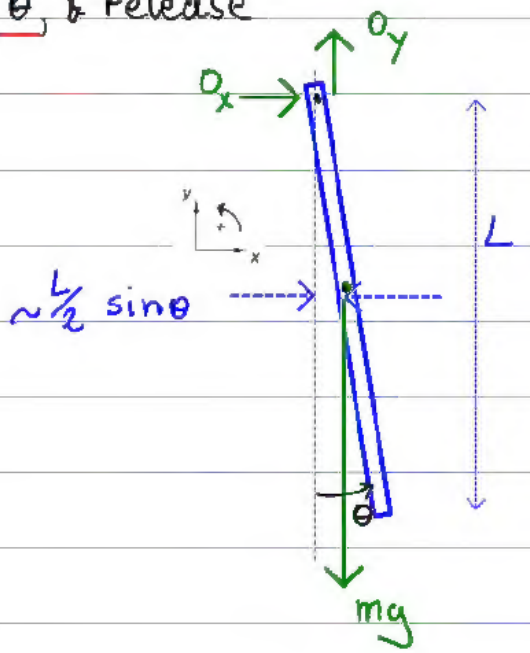
1/ Define $\theta = 0$ as neutral position of pendulum.

2/ Define +ve direction for rotation

3/ Give pendulum a small +ve deflection θ, & release

4/ $\sum M_o = -(mg)\left(\frac{L}{2}\sin\theta\right) = I_o \alpha$

Pendulum has a fixed axis



5/ $-mgL\theta/2 = \left(\frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2\right)\ddot{\theta}$

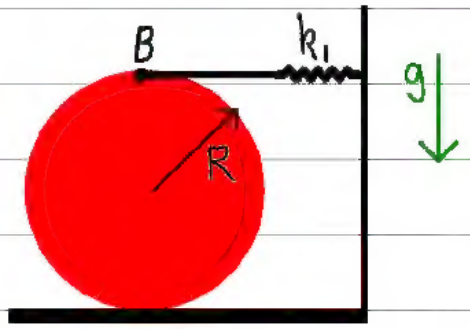
6/ $\frac{mL^2}{3}\ddot{\theta} + \frac{mgL}{2}\theta = 0$

7/ $\ddot{\theta} + \frac{3g}{2L}\theta = 0$

8/ $\ddot{\theta} + \omega_n^2 \theta = 0$, where $\omega_n = \sqrt{3g/2L}$; $T = \frac{2\pi}{\omega_n}$

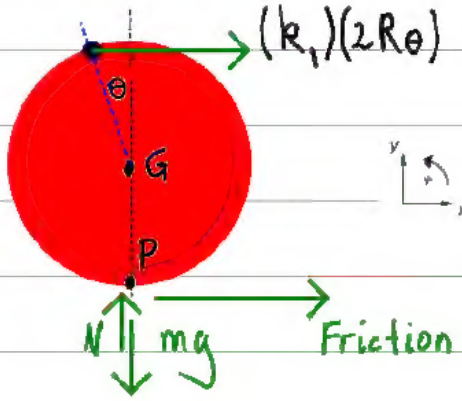
9/ $\theta(t) = \text{Amplitude} \cdot \sin(\omega_n t + \phi)$

Eg.



Given: wheel radius $R = 0.6m$, no slip
radius of gyration $k = 0.4m$
wheel mass $m = 22kg$
spring stiffness $k_1 = 1200\ N/m$
wheel is in equilibrium at position shown

Find ω_n



1/ FBD, axes, selection of origin

2/ Roll wheel a small +ve θ, & release

3/ spring is stretched by $2R\theta$

4/ spring force is $2R\theta k_1$

5) $\Sigma M_G = I_G \alpha$?
$\Sigma M_P = I_P \alpha$? ★
$\Sigma M_O = I_O \alpha$?

6/ $\Sigma M_P = I_P \alpha$

$$-(2R\theta k_1)(2R) = (mk^2 + mR^2)\ddot{\theta}$$

$$-(2)^2(0.6)^2(1200)\theta = \left[(22)(0.4)^2 + (22)(0.6)^2\right]\ddot{\theta} = 0$$

$$-1728\theta = 11.44\ddot{\theta}$$

$$\ddot{\theta} + 151\theta = 0 \longleftrightarrow \ddot{\theta} + \omega_n^2\theta = 0$$

$$\rightarrow \omega_n = \sqrt{151} = 12.3 \text{ radians/s} = 1.96 \text{ Hz}$$

$$\rightarrow \theta(t) = C_3 \sin(12.3t + \phi)$$

- Real vibrations lose energy through various mechanisms, e.g.
  - friction
  - wind resistance
  - heat generation via spring deformation
  - etc.

- We can adopt an approximate model for a specific energy loss mechanism.

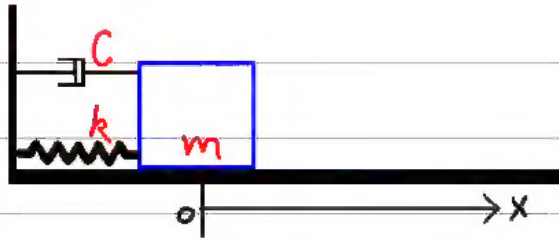
- We will model energy loss due to 'viscous damping'

  e.g. the vibration occurs in a viscous fluid, or there is cyclic plastic def motion.

We introduce the "damper" or "dashpot", which exerts a force proportional to velocity, in opposite direction.

Symbol C: $\left(\frac{N}{m/s}\right)$

$$\Sigma F_x = -kx - c\dot{x}$$

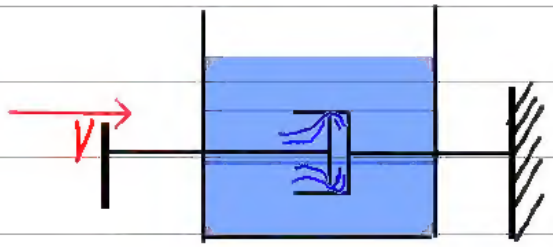


- Note the damper has no effect if the mass is not moving.

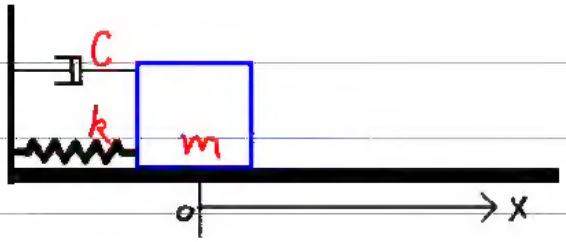
- The arrangement & strength of springs determines location of neutral point (not the dampers)

Damper / Dashpot

viscous fluid must leak past edges of piston as it is inserted or retracted



→ viscous damping force
- proportional to velocity
- in opposite direction to velocity



problem solution:

1/ Set origin at neutral point (damper has no effect)
Select positive direction

2/ Displace mass a small +ve distance x, and give it a small +ve velocity $\dot{x}$. Release the mass.

3/ $\sum F_x = -kx - c\dot{x} = m\ddot{x}$

4/ Put eqn in standard form:

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0 \quad , \text{ where } \omega_n^2 = k/m$$

Solution in Next Lecture!

Summary

Basic Linear Vibrations

Translational

$$\sum F_x = -kx = ma_x = m\ddot{x}$$

$$\ddot{x} + \frac{k}{m}x = 0$$

$$\ddot{x} + \omega_n^2 x = 0, \quad \omega_n^2 = k/m$$

$$x(t) = \text{Amplitude} \cdot \sin(\omega_n t + \phi)$$

Rotational

$$\sum M_o = -K\theta = I_o\alpha = I_o\ddot{\theta}$$

$$\ddot{\theta} + \frac{K}{I_o}\theta = 0$$

$$\ddot{\theta} + \omega_n^2\theta = 0, \quad \omega_n^2 = K/I_o$$

$$\theta(t) = \text{Amplitude} \cdot \sin(\omega_n t + \phi)$$

- Rotating vibration examples: pendulum, rolling wheel
- Introduction to viscous damping

Next Lecture

- Damped Vibrations (Meriam 8/2)